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U.S. ARMY INTELLIGENCE CENTER AND SCHOOL SOFTWARE ANALYSIS AND MANAGEMENT SYSTEM

THE NEED TO TRACK SAMPLE SIZE IN CONJUNCTION WITH MEMO No. 72, DATED 04/01/86

TECHNICAL MEMORANDUM No. 12

MARC

Mathematical Analysis Research Corporation



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20. ABSTRACT (Continue on reverse olds if necessary and identify by block number)

This memo extends the results for Tech Memo # 11. By use of simulation, two EEP ellipses, one or both inappropriately (by use of the wrong statistic) normalized, are combined. The probability (obtained by simulating the ellipse combination 1000 times) of the resultant ellipse containing the target is then compared to the theoretical result.

U.S. ARMY INTELLIGENCE CENTER AND SCHOOL Software Analysis and Management System

The Need To Track Sample Size In Conjuction With Memo No. 72, Dated 04/01/86

Technical Memorandum No. 12

02 April 1986

Author:	
Michael W Rennie	Lone Myhre
Mathematical Analysis Res	search Corporation

Approval:	Dlung Akeroid
Tames W. Gillis, Subgroup Leader Algorithm Analysis Subgroup	Edward J. Records, Supervisor USAMS Task
2. 2	

A. F. Ellman, Manager Ground Data Systems Section

Fred Vote, Manager Advanced Tactical Systems Accessor for

NESS CHARLE

DATE CHARLE

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JET PROPULSION LABORATORY California Institute of Technology Pasadena, California

PREFACE

The work described in this publication was performed by the Mathematical Analysis Research Corporation (MARC) under contract to the Jet Propulsion Laboratory, an operating division of the California Institute of Technology. This activity is sponsored by the Jet Propulsion Laboratory under contract NAS7-918, RE182, A187 with the National Aeronautics and Space Administration, for the United States Army Intelligence Center and School.

The Need To Track Sample Size (In Conjunction With Memo No. 72, Dated 04/01/86)

ATTACHMENTS: APPENDICES I, II, III.

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The role of sample size (number of Lines Of Bearing) depends on the set of algorithms one uses. Some mixes of algorithms can produce undesirable results because of the way sample size is handled (or not handled.) The most obvious place to look for this source of problems is mismatching of the Chi-square and F distributions. In this memo we examine two issues:

- (1) How do ellipses based on F-distributions combine with other ellipses? The theory for ellipse combination is based on the ellipses having known covariance and hence Chi-square cut-off values.
- (2) What is the impact of converting F confidence levels as if they were Chi-square?

The conclusion of this memo is that both of the sources of error work in the same direction, namely making the resultant error ellipse size too small to have 95% confidence of capturing the emitter. The smaller the sample sizes of the reported ellipses the greater the problem. Furthermore, corrections for these errors require knowledge of sample size.

The two issues given above are discussed in more detail in the sections that follow. Appendices with simulation results and showing size variation follow.

I. COMBINATION OF ELLIPSES WITH ESTIMATED COVARIANCE

MARC used simulation to study this problem. The simulation used and results found will be discussed at the end of this section. With hindsight it is possible to explain why the results turned out the way they do. We present this explanation in question-answer format before showing the simulation results:

(i) What is the most significant difference between the ellipse that combination theory calls for and the ellipse the F-test generates?

Answer: The theory assumes that location error is being accounted for. With small sample size, however, ellipse size estimates vary so much that ellipse size error dominates.

(ii) Isn't this difference also an issue before ellipse combination?
Answer: Yes. The method of handling it is to make the ellipse bigger. By making it enough bigger one can regain 95% confidence.

(iii) If bigger ellipses are being combined won't the result be bigger, enough bigger to still have 95% confidence?

Answer: The combined ellipse is certainly bigger than it would be if the source ellipses were not based on the F. Our simulations suggest that this is not quite big enough. The problem is that we are exposed to a 5% risk of having any individual incoming ellipse be too small. If two ellipses are being combined then this a total of two 5% risks or almost a 10% risk.

(iv) Does having one of the source ellipses too small always cause a problem?

Answer: No, not always. Simulation yielded about 92% correct or 8% in error in our example. The reason it is a problem most of the time is probably that when an incoming ellipse is smaller than it should be it is weighted heavily in determination of resultant location and error ellipse size. The 'bad apple' becomes the driver in the problem.

(v) How do you know that the difference between 8% error observed and 5% error expected doesn't reflect some other problem such as bias?

Answer: Two methods were used to guarantee that the correct problem was being isolated. First a 'controlled experiment' or simulation was run with the Chi-square. Second and more importantly, two simulations were run for the same sensor-emitter setup with only sensor accuracy changed. It is easily shown that bias and other effects are a second order effects. To study behavior uninfluenced by them it is sufficient to make sensor accuracy good enough. To guarantee this MARC first performed the simulation with a one degree standard deviation. Then to prove this was small enough a second simulation was performed with a .1 degree standard deviation.

(vi) How do you know that you weren't using cases that the acceptance test would have rejected?

Answer: This would still be a problem although admittedly a different one. MARC intends to investigate this further. Our belief is, however, that study will show that this type of rejection will only occur when either

- a) two ellipses are too small
- or b) one is too small and the another is naturally small. The simulations primarily reflect the case of one ellipse too small and the other ellipse being reasonable sized as this is more likely in our problem.

A description of the individual simulations follows.

1000 runs each.

Emitter Location: (20,20)

Ellipse 1 is based on 5 LOBs from (0,0),(5,0),(10,0),(15,0),(20,0).

Ellipse 2 is based on 5 LOBs from (10,0),(15,0),(20,0),(25,0),(30,0).

(Note although the locations used overlap different LOBs were generated.)

Fix Method: (Weighted) Perpendicular with an estimated variance on angular measurements using the F-distribution.

95% Cut-off level for the individual ellipses (based on the F)=19.1 Percent of combined ellipses capturing the emitter using a 19.1 cutoff

- i) 91.9% when the standard deviation = ! degree
- ii) 91.9% when the standard deviation = .1 degree (note that this agreement is a coincidence)

Cutoff that yields a 95% chance of capturing the emitter

- i) 27.9 when the standard deviation = 1 degree
- ii) 29.8 when the standard deviation = .1 degree

 Note that the square root of 27.9/19.1 and 29.8/19.1 yield the percentage of further growth in the ellipse size (radii length) to have a 95% chance of capture. Computing these equal 1.209 and 1.249 respectively.

A number of graphed examples of the ellipses corresponding to a sequence of the 1 degree runs are included in the appendix. A listing of the cutoff scores is also included. (These are the numbers that should be compared with 19.1 for the F cutoff or 5.99 for the Chi-square.)

II. IMPROPER CONVERSION

Suppose that a fix source is reporting to a fusion center. Further suppose that the source is reporting an ellipse based on

i) 5 LOBs

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ii) a 50% F cut-off value appropriate for sample size 5 The fusion center would report an ellipse that is 1.59 times smaller on each axis than it should be.

The method of arriving at this is to note that the Chi-square conversion to a 95% ellipse assumes that the cutoff level must go from 1.39 to 5.99. In fact it must go from 1.76 to 19.1. Thus the Chi-square conversion process scales up ellipse size by the square root of 5.99/1.39 or 2.075 when it should scale up by the square root of 19.1/1.76 or 3.29. (1.59=3.292/2.075)

The incorrectly converted cut off value would be 1.76*5.99/1.39*7.58. Note that in the simulations discussed in the previous section this would only have captured the emitter

- i) 77.9% of the time when the standard deviation = 1 degree
- ii) 79.0% of the time when the standard deviation = .1 degree

APPENDICES I

Sorted cut-off level from 1000 simulations with angular standard deviation of one degree.

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Û

APPENDICES II

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APPENDICES III

A sequence of ellipse from the 1 degree simulation to show the variability of ellipse size.

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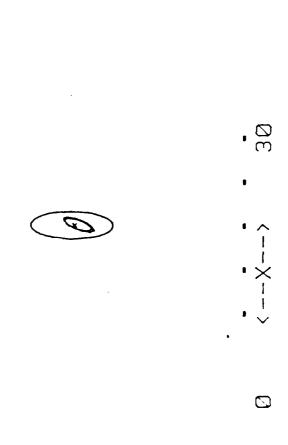
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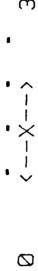
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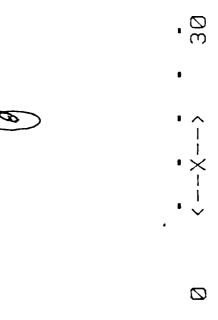
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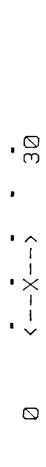
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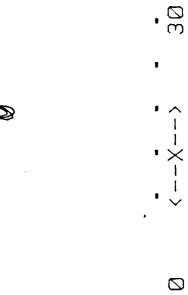




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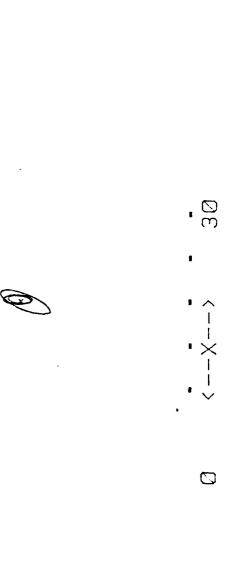
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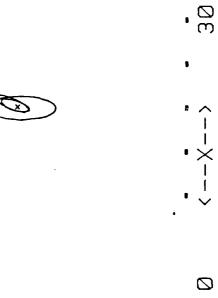
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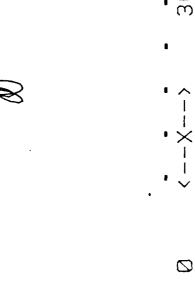
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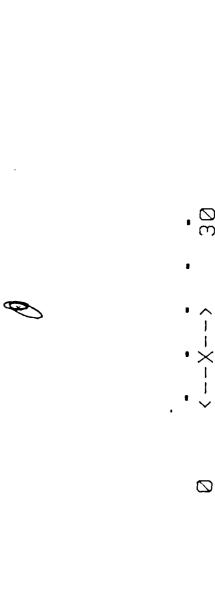


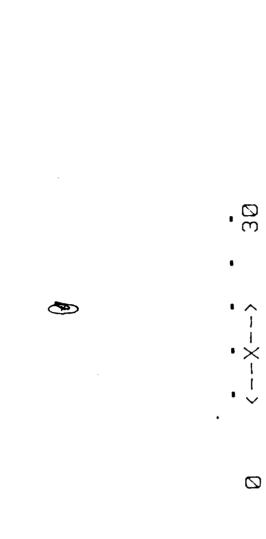


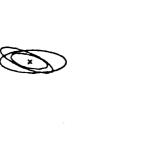
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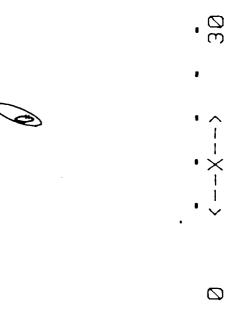








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